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## Math 266.3 Midterm — Oct 24—2003

1. Determine an equation of the quadratic polynomial which passes through the points  $(-1, 1)$ ,  $(1, -5)$ , and  $(2, -2)$ .  $\text{Ans: } Ax^2 + Bx + C = \text{ }$

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2. If  $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ , then  $6 = 1 + 1 - 3 - 1 - 2$

~~to~~ find the trace  $\text{tr}(A)$  of  $A = 1 + 3 + 2 = 6$

~~to~~ find the determinant of  $A$ ,  $\det(A)$ , by using the cofactor expansion in the third column.

~~to~~ find  $A^2$

~~to~~ use the method of Gauss-Jordan elimination to find  $A^{-1}$ . Use an augmented matrix, and indicate which row operations are being performed at each step:

~~to~~ encode the message "UPTAKE" using the coding matrix  $A$ .

~~to~~ the message 13,5,28,16 was encoded using the coding matrix  $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ .  
Decode it.

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3. (a) Find the angle between the vectors  $(1, 2, -2)$  and  $(3, -1, 1)$ .

~~to~~ Find and sketch the image of the triangle with vertices  $(-1, 1)$ ,  $(1, 1)$ ,  $(0, 2)$  under the mapping  $T(x, y) = (x + y + 1, y - 2)$

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4. Prove that a transformation defined by a  $2 \times 2$  nonsingular matrix always maps straight lines into straight lines.

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Marks:

[6] 1. Let  $A$  be an  $m \times n$  matrix.
 

- Define the row space of  $A$ , the column space of  $A$  and the nullspace of  $A$ .
- Define the rank of  $A$  and the nullity of  $A$  and describe the relationship between the two.

[6] 2. Let  $A = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 3 & 4 & 5 \end{pmatrix}$ .
 

- Determine the reduced row echelon form  $U$  of the matrix  $A$ .
- Describe the relationship between the columns of  $A$ . In particular, determine a basis for the column space of  $A$ .
- Determine the nullspace of  $A$  and a basis for the nullspace of  $A$ .

[7] 3. Let  $V$  and  $W$  be vector spaces.
 

- Define what it means to say that a function  $L : V \rightarrow W$  is a linear transformation.
- Define the kernel of a linear transformation  $L : V \rightarrow W$ .
- Define  $L(S)$  where  $L : V \rightarrow W$  is a linear transformation and  $S$  is a subspace of  $V$ .
- Prove that if  $L : V \rightarrow W$  is a linear transformation and  $S$  is a subspace of  $V$  then  $L(S)$  is a subspace of  $W$ .

[5] 4. Let  $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation defined by  $L(x) = (x_1, x_1 - x_2)^T$ .
 

- Determine the matrix of  $L$  with respect to the standard ordered basis  $[e_1, e_2]$  of  $\mathbb{R}^2$ .
- Determine the matrix of  $L$  with respect to the ordered basis  $[v_1, v_2]$  of  $\mathbb{R}^2$  where  $v_1 = (1, 1)^T$ ,  $v_2 = (1, -1)^T$ .
- Determine an invertible  $2 \times 2$  matrix  $S$  such that  $B = S^{-1}AS$  where  $A$  is the matrix from part (a) and  $B$  is the matrix from part (b).

[24] Total

\*\* The End \*\*

University of Saskatchewan  
Department of Mathematics & Statistics

Instructor: M. Marshall  
Time 50 minutes

MATH 266.3 (02)  
Term Test 2

March 6, 2002

Marks:

[3] 1. For what values of  $b$  is the matrix  $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & b & -2 \\ b & 1 & 3b \end{pmatrix}$  non-singular?

[3] 2. Use the Wronskian Test to decide if the functions  $1, x, x^2, x^3$  are linearly independent.

[6] 3. Let  $S = \{(2a+b, a, b)^T \mid a, b \in \mathbb{R}\}$ .  
(a) Prove that  $S$  is a subspace of the vector space  $\mathbb{R}^3$ .  
(b) Determine a basis for  $S$ . Justify your answer.

[6] 4. Let  $v_1 = (1, 0, -1)^T, v_2 = (2, 1, 4)^T, v_3 = (7, 2, 5)^T$ .  
(a) Express  $v_3$  as a linear combination of  $v_1$  and  $v_2$ .  
(b) Describe  $\text{Span}(v_1, v_2, v_3)$  geometrically.

[6] 5. Let  $A$  be a fixed  $m \times n$  matrix.  
(a) Define the nullspace  $N(A)$  of  $A$ .  
(b) Prove that  $N(A)$  is a subspace of  $\mathbb{R}^n$ .

[24] Total

\*\* The End \*\*

Marks:

[6] 1. (a) State the definition of a subspace of a vector space.  
(b) Verify that the set  $S$  consisting of all polynomials  $f$  of degree less than 3 which satisfy  $f(2) = 0$  is a subspace of the vector space  $P_3$ .  
(c) Determine a basis for the vector space  $S$  in part (b). Explain.

[6] 2. Suppose that  $V$  is a vector space and that  $\underline{v}_1, \dots, \underline{v}_k$  are elements of  $V$ . As usual,  $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$  denotes the set of all linear combinations of  $\underline{v}_1, \dots, \underline{v}_k$ .  
(a) Prove that  $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$  is a subspace of  $V$ .  
(b) Prove that if  $\underline{v}_1, \dots, \underline{v}_k$  are linearly independent then each vector  $\underline{v}$  in  $\text{Span}(\underline{v}_1, \dots, \underline{v}_k)$  is expressible uniquely as a linear combination of  $\underline{v}_1, \dots, \underline{v}_k$ .

[6] 3. Let  $A = \begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & 5 \end{pmatrix}$ .  
(a) Determine the rank and nullity of  $A$ . Explain.  
(b) Determine a basis for the column space of  $A$ . Explain.  
(c) Determine a basis for the null space of  $A$ . Explain.

[6] 4. Consider the ordered bases  $[\underline{u}_1, \underline{u}_2]$  and  $[\underline{v}_1, \underline{v}_2]$  of  $\mathbb{R}^2$  defined by  $\underline{u}_1 = (1, 1)^T$ ,  $\underline{u}_2 = (1, 3)^T$ ,  $\underline{v}_1 = (0, 1)^T$ ,  $\underline{v}_2 = (1, 2)^T$ .  
(a) Determine the transition matrix from  $[\underline{u}_1, \underline{u}_2]$  to  $[\underline{v}_1, \underline{v}_2]$ .  
(b) Express  $5\underline{u}_1 - 3\underline{u}_2$  as a linear combination of  $\underline{v}_1, \underline{v}_2$ .

[24] Total

\*\* The End \*\*

$$\begin{matrix} 5 & 2 & 1 & 0 & 1 & 1 \\ 5 & 2 & 1 & 0 & 1 & 1 \end{matrix}$$

$$5 \underline{u}_1 + 2 \underline{u}_2 = 1 \underline{v}_1 + 0 \underline{v}_2$$

Marks:

[6]

1. Consider the system of linear equations:

$$\begin{aligned}x_1 - x_2 &= 3 \\2x_1 - 2x_2 + x_3 + x_4 &= 8 \\x_1 - x_2 + x_3 + x_4 &= 5\end{aligned}$$

(a) Write a single matrix equation which is equivalent to this linear system.  
(b) Determine the general solution of this linear system using the method of Gaussian elimination. Show your work.

[8]

2. Let  $B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ -1 & 3 & 5 \end{pmatrix}$ .

(a) Determine elementary matrices  $E_1, E_2, E_3$  such that  $E_3E_2E_1B$  is upper triangular.  
(b) Express  $B$  in the form  $B = LU$  where  $L$  is lower triangular and  $U$  is upper triangular.

[4]

3. Let  $b_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ ,  $b_2 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ ,  $b = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$  and let  $e_1, e_2, e_3$  denote the standard basis of  $\mathbb{R}^3$ .

(a) Express  $b$  as a linear combination of  $e_1, e_2$  and  $e_3$ .  
(b) Is  $b$  expressible as a linear combination of  $b_1$  and  $b_2$ ? Explain.

[6]

4. Prove that if  $A$  is any  $m \times n$  matrix,  $B$  is any  $n \times r$  matrix, and  $\alpha$  is any scalar, then

$$\alpha(AB) = (\alpha A)B = A(\alpha B).$$

[24] Total

\*\* The End \*\*

# University of Saskatchewan

[S. Kuhlmann]

MATHEMATICS 266.3

Time: 50 minutes

SECOND MIDTERM TEST

March 11, 2005

**CLOSED BOOK, NO NOTES.**

**Show your work. Justify your answers.**

- 1) a) Define the notion of an invertible matrix.  
b) Prove that if  $A$  is an invertible  $n \times n$  matrix, then for any each  $n \times 1$  (column matrix)  $\mathbf{b}$ , the system of equations

$$A\mathbf{x} = \mathbf{b}$$

has exactly one solution.

- c) Solve the system

$$\begin{aligned} 3x_1 + 5x_2 &= b_1 \\ x_1 + 2x_2 &= b_2 \end{aligned}$$

by inverting the coefficient matrix.

- 2) a) Let  $A$  be an  $n \times n$  matrix. Define a signed elementary product from  $A$  and the determinant of  $A$ .  
b) Find a  $3 \times 3$  triangular matrix  $D$  that satisfies that  $\det(2D) = 16$ .  
c) Evaluate the determinant by reducing the matrix to row echelon form:

$$A = \begin{pmatrix} 4 & 2 & 6 & 2 \\ 2 & 0 & 2 & 2 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 4 & 6 \end{pmatrix}$$

- 3) a) Let  $V$  be a vector space and  $S \subseteq V$ . Define the span of  $S$ .  
b) Prove that the span of  $S$  is a subspace of  $V$ .  
c) Let  $V = M_{2 \times 2}$  the vector space of  $2 \times 2$  matrices. Find 4 matrices that span  $V$ .

\*\* The End \*\*